stall flutter design system. Such an analysis must properly account for all the parameters important to aeroelastic stability before durable rotors can be confidently designed with minimum weight and performance penalties. Efforts at Pratt & Whitney Aircraft and the United Aircraft Research Laboratories to provide an accurate and dependable design system have recently been intensified. Major emphasis is being concentrated on developing an energy method of stability analysis. Attempts to generate the unsteady stalled cascade aerodynamics for the analysis include both theoretical investigations and experimental measurement of unsteady lift and moment in an oscillating cascade rig. The blade mode shape prediction is also being improved. NASTRAN is being developed for determining dynamic chordwise deflection, and shroud models are being improved to account more accurately for stiffness and boundary conditions (slippage). The ultimate goal of this work is a system that extends the state-of-the-art to make stall flutter a problem that can be successfully handled during preliminary fan and compressor design.

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Engineering Note

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Modeling of Gas Turbine Engine Compressor Blades For Vibration Analysis

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Introduction

THE vibration analysis of gas turbine engine blades is done prior to engine buildup to preclude the possibility of resonance problems in the fundamental modes. Experimental testing is not always feasible especially in the design phase of development as hardware is either nonexistant or unavailable. Several analytical procedures have been developed to predict blade frequencies of which the lumped mass approach and more recently finite element analysis are the most popular. The engineer has the option to choose which method will best suit his needs. The lumped mass approach can give acceptable results using little computer facilities, but only on a special class of blades. Finite element theory on the other hand will give acceptable results on all types of blades, but requires large computer resources and considerable computer time. This paper will examine these two methods of vibration analysis with respect to modeling procedure and validity of results.

Discussion

Vibration analysis using the lumped mass approach is severely limited by the nature of the model. The blade is modeled using chordwise segments lumped to discrete points at the center of the segment. Each segment is considered a point mass and elastic beam with stiffness properties and segment stagger angle taken as constant between masses with these values a representative average of the physical segment. The size of the segments are determined to best simulate the blade by requiring the segment to have relative constant physical geometry. A Holzer-Myklestad approach was used for eigenvalue extraction.

Reference 1 describes the analysis procedure in detail. The idealization this lumped mass approach gives cannot take into account space irregularities such as camber and ramp angle because of the two-dimensional limitations of beam theory. Thus the types of blades analyzed using this procedure is limited to low camber, high aspect ratio blades.

NASA Structural Analysis (NASTRAN) program was used to study the finite element modeling of blades as this program is coming into rather wide use. All modeling was done using a four-noded plate element (CQUAD) or a three noded triangular plate element (CTRIA). These elements have both inplane and bending stiffnesses that assumes a solid, constant thickness, homogeneous cross section. Energy theory applied to a polynomial distribution of displacement is used for the calculations. Transverse shear flexibility is included. Reference 2 provides a detailed theoretical discussion of the inverse power method of eigenvalue extraction used and the finite elements used in the analysis. Reference 3 describes the analysis procedure for the normal mode analysis.

To determine a correct modeling procedure for each method of analysis, flat plates with various aspect ratios were analyzed using an iterative modeling procedure. For the lumped mass analysis, the plates were modeled with an increasing number of constant property segments. Figures 1 and 2 illustrates how the results converge to a natural frequency using increasing number of segments. This would indicate the accuracy of the frequencies is a function of degrees of freedom of the model. This is verified by

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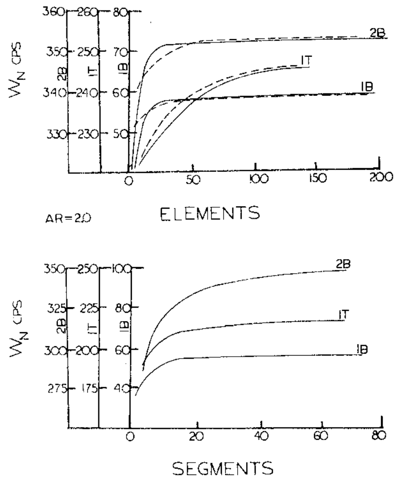

Fig. 1 Convergence of a cantilevered plate to its natural frequencies. t = 0.0625 in., b = 3.0 in., l = 6.0 in. The number of triangular plate elements are halved for scaling and are represented by the dash lines.

examining the longer convergence rates required for the higher modes. The bulk of finite element analysis was accomplished using the CQUAD2 element. An increasing number of quadrilateral elements were used for the mesh. The aspect ratio of the elements were kept as close to one as practical to preclude differences in the analysis due to high aspect ratio elements. A plate was also modeled with triangular plate elements (CTRIA2) for comparison to the convergence rates of the quadrilateral plate element analysis. Figures 1 and 2 show the convergence rates for this study. As in the lumped mass analysis, the convergence rates are longer for the higher modes of vibration. The asymptotic frequencies obtained were compared to the natural frequencies predicted by plate theory. Reference 4 was used for the theoretical predictions with the results shown in Table 1. Figure 3 illustrates the accuracy to which the lumped mass and finite element calculated the first and second bending and first torsional modes. Note on the lumped mass analysis, the difference with the lower aspect ratio plates. This is a result of beam theory used in the lumped mass analysis trying to predict frequencies as a plate. As the aspect ratio of the plate increases, the more closely a beam is approximated and, thus a more accurate analysis. The accuracy obtained in the finite element analysis is good. The slight variation in the higher aspect ratio second bending mode is due primarily to the variation in the experimental results for the frequency parameters used in the calculation of the theoretical results (Ref. 4, p. 80).

Upon examination of the results of the study, one can determine the type of analytical procedure to use as well as the complexity of the model. To demonstrate this, two

Table 1 Theoretical frequencies from Ref. 4: Aluminum plate t = 0.0625 in., b = 3.0 in.[a]

AR	1B	1T	2B
1	234.	572.	1436.
2	56.	237.	356.
5	9.24	57.7	93.
8	3.52	22.0	52.0

[a] All frequencies in cps.

Table 2 Comparison of analyses to holographic analysis: High aspect ratio blade[a]

Analysis	1 Mode	2 Mode	3 Mode
NASTRAN	1040	3977	4585
Lumped mass	945	3762	4256
Holography	978	3911	4557

[a] All frequencies in cps.

Table 3 Comparison of analyses to holographic analysis: Low aspect ratio blade[a]

Analysis	1 Mode	2 Mode	3 Mode
NASTRAN	523	1608	2176
Lumped mass	539	1576	2105
Holography	491	1109	2040

[a] All frequencies in cps.

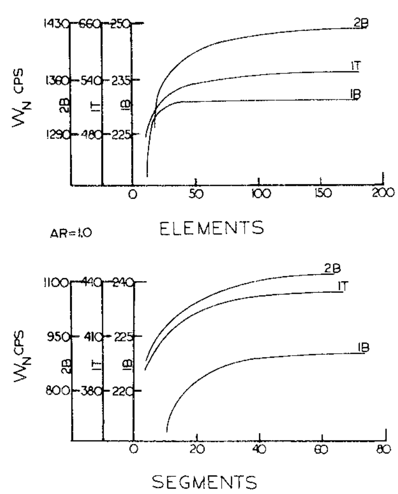

Fig. 2 Convergence of a cantilevered plate to its natural frequencies; t = 0.0625 in., b = 3.0 in., l = 3.0 in.

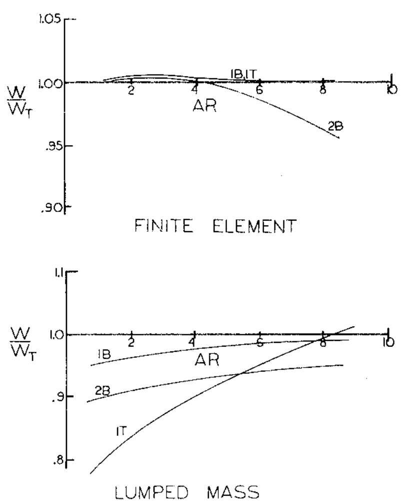

Fig. 3 The effect aspect ratio has on the convergence of calculated frequencies to theoretical frequencies of a cantilevered plate.

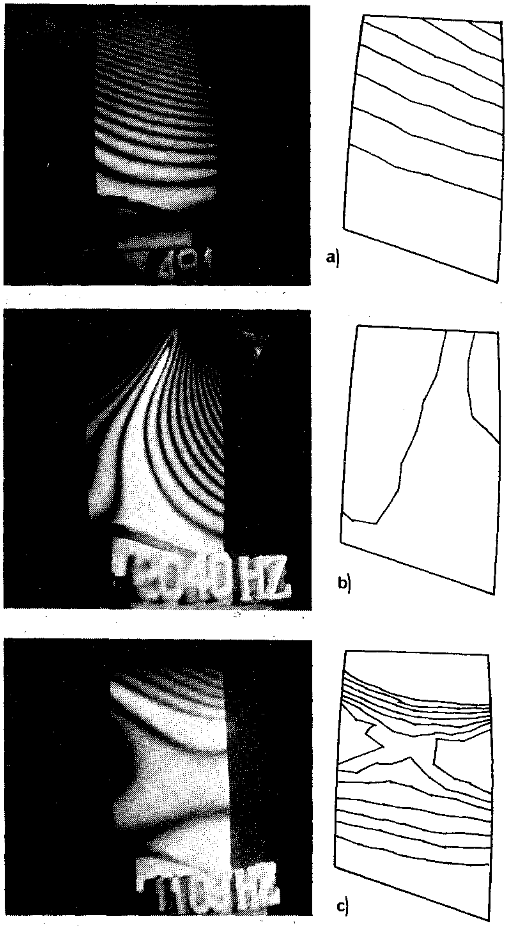

Fig. 4 Comparison of holographic mode shapes to those calculated by NASTRAN. a) 1 bending, b) 1 torsion, c) complex mode.

compressor blades were analyzed using the lumped mass and then the finite element approach. Natural frequencies for the blades were determined using holographic analysis for comparison. Each blade was modeled using 25 constant property segments for the lumped mass analysis. The finite element analysis was done using 100 triangular plate elements. Triangular plate elements were chosen because of their inherent planar configuration in space. Tables 2 and 3 summarize the results of this study. In addition to calculating the natural frequencies for these blades, finite element theory can also determine mode shape because of its three dimensional capability. Figure 4 shows the comparison of the modes shapes determined from the finite element procedure and that obtained using holographic analysis.

Conclusion

Because of the geometric limitations imposed by gas turbine engine blade, a three dimensional analysis is essential. The results show however that for high aspect ratio compressor and fan blades, acceptable results can be obtained with a great savings in computer resources with a two-dimensional lumped mass approach. The results also show that a blade can be over modeled. The fast convergence of the finite element models and high aspect ratio lumped mass models verify that fact. Considering computer resources are at a premium, an optimum model is essential.

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